Exercise 155

For the following exercises, solve the trigonometric equations on the interval $0 \leq \theta < 2\pi$.

$$2\sin\theta - 1 = 0$$

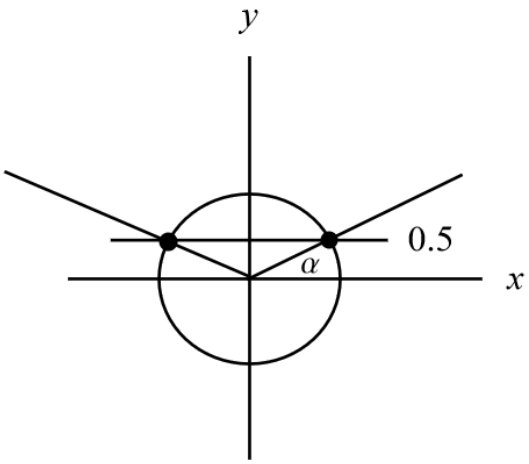
Solution

$$2\sin\theta - 1 = 0$$

$$2\sin\theta = 1$$

$$\sin\theta = \frac{1}{2}$$

We want the two angles to the points on the unit circle that are a distance $1/2 = 0.5$ up.



Taking the inverse sine of $1/2$ gives 30°, or $\pi/6$ radians. This is α in the figure.

$$\alpha = \frac{\pi}{6}$$

To obtain the counterclockwise angle from the positive x-axis to the second point, subtract this angle from π.

$$\pi - \alpha = \frac{5\pi}{6}$$

Therefore,

$$\theta = \left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}.$$